



Mathematics and Science Braille

Nemeth in UEB

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Contents

Introduction	4
Basic Format	5
Special Symbols	6
Use of Opening and Closing Nemeth Indicators.....	7
Example 1.....	7
Example 2 (displayed)	8
Basic Guidance on When to Switch	10
Example 3.....	10
Exceptions	11
Example 4 (numbers)	11
Example 5 (letters and numbers).....	12
Example 6 (exception to the exception).....	13
Example 7 Mathematical Expressions and Free Standing Numbers	14
Example 8 (punctuation/single-word switch).....	15
Example 9 (punctuation/single-word switch).....	16
Basic Guidance (continued)	18
Example 10 (math symbols)	18
Additional Guidelines.....	19
Example 11 (percent)	20
Example 12 (Greek letters, degrees)	22
Example 13 (words in equations).....	23
Example 14 (measurement units)	24
Example 15 (identifiers)	26
Example 16 (spatial).....	28
Example 17 (directions).....	32
Example 18 (comments).....	33
Example 19 (comments).....	34
Example 20 (one-word comment).....	35
Example 21 (emphasis/labeled statement)	37
Example 22 (partial emphasis).....	38
Example 23 (placement of switches).....	39
Example 24 (placement of switches).....	40

Example 25(embedded matrix)	42
Example 26 (table)	43
Example 27 (table)	44
Example 28 (table)	45
Switches and Formatting	46
Full Pages from Math Textbooks	55

Introduction

We are going to explore sections of the guidance in the order laid out in the document posted on the website of Braille Authority of North America (BANA).

The first and overriding principle is that, within a UEB document, mathematical and technical expressions will be transcribed in Nemeth Code. This is accomplished by setting apart the technical material with the use of the Opening Nemeth Indicator and the Nemeth Terminator. **No contractions are permitted within these switches.** Text outside of the switches is transcribed in UEB.

The single-word switch indicator was created to allow contractions in a single word between math expressions and to avoid having to switch out of Nemeth code for one word. The switch is required on a single word even if the word contains no contractions. The effect of the single-word switch is terminated by a space. (The single-word switch indicator is a Nemeth symbol, not a UEB symbol.)

Opening Nemeth Code indicator ⠠⠠

Nemeth Code Terminator ⠠⠠

Single-word switch indicator ⠠⠠

(The dot locator for mention ⠠⠠ precedes each indicator in the list on the Special Symbols page.)

Basic Format

The switch indicators signal that the symbols used are Nemeth Code symbols, but the switches do not govern formatting. The entire document is formatted according to the established mix of *Nemeth Code* and *Braille Formats* provisions as posted on the website of Braille Authority of North America and available there for downloading. When the technical material is transcribed in Nemeth Code, the whole document is formatted according to the Nemeth Code--even the parts of the text that are not in Nemeth. We will discuss format later in the session.

Special Symbols

When material is transcribed using the Nemeth Code, and this is stated on the Transcriber's Notes page, it is not necessary to list on the Special Symbols page any symbol of the Nemeth Code. The TN page should have a note stating: "Mathematical content is transcribed according to the Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision, 2007-2015 Updates, including the guidelines for using Nemeth in a UEB context."

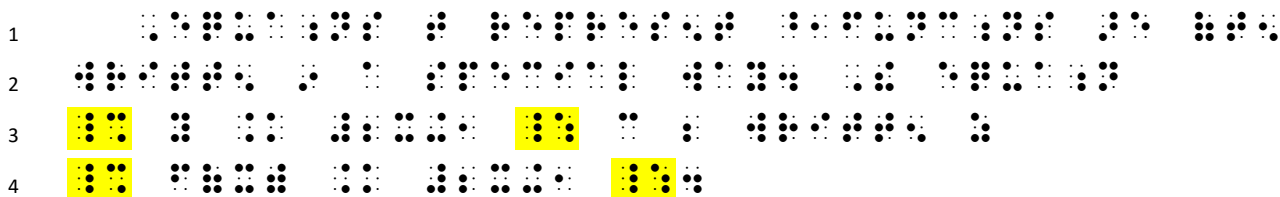
However, the single-word switch indicator is a new symbol and may not be recognized by the reader. It should be listed on the Special Symbols page.

Use of Opening and Closing Nemeth Indicators

Place the opening Nemeth Code indicator *followed* by a space before the sequence to which it applies. Its effect is terminated by the Nemeth Code terminator *preceded* by a space. The spaces required with the indicator and terminator do not represent spaces in print.

Example 1

Equations that represent **functions** are often written in a special way. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$.



Line 1: All paragraphs in a document governed by Nemeth formatting begin in cell 3 with runovers in cell 1. There are no blocked paragraphs in Nemeth.

Line 1: UEB boldface word indicator is used on a bold word that is not within the Nemeth Code indicators.

Lines 1 and 2: Note the contraction for "tion" in the word equation

Line 3, 4: Nemeth Code is initiated by the use of the Opening Nemeth indicator and ends with the Nemeth Terminator. Math expressions are transcribed according to the rules of *The Nemeth Code*, within the Nemeth indicators. If possible, the indicators are on the same line as the expression.

Line 4: The period applies to the structure of the sentence and not to the math expression. It follows the termination indicator.

When the Nemeth Code text is displayed on one or more lines separate from the UEB text, the Opening Nemeth indicator may be placed on a line by itself or at the end of the previous line of text; the terminator may be placed at the end of the last line of the math material or on a line by itself. The opening indicator is placed at the end of the text line preceding the math expression in preference to being on a line by itself.

Example 2 (displayed)

The factor by which the sound intensity must be increased to double the loudness can be determined by the method used in Example 9:

$$\beta_2 - \beta_1 = 10.0 \text{ dB} = (10 \text{ dB}) \left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right) \right]$$

Solving this equation reveals that $I_2/I_1 = 10.0$.

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Braille representation of the equation and text above. The opening Nemeth indicator is on line 3, and the terminator is on line 8. The opening indicator is placed at the end of the text line preceding the math expression in preference to being on a line by itself.

Line 1: Indented paragraph according to Nemeth format.

Line 3: Single number is transcribed in UEB.

Line 3: Opening Nemeth Code Indicator is placed at the end of the text line preceding the displayed material.

Line 4-7: Displayed expression. A division must be made at every comparison sign because a division is necessary within a link.

Line 8: The Nemeth Code Terminator will not fit at the end of the last line of the displayed material. It is placed on the following line in the runover position.

Line 10: The entire technical expression and the code indicators will fit on one line. The period is related to the sentence structure, not to the math expression.

When punctuation that logically applies to the sentence structure appears between technical expressions, the punctuation should be transcribed in Nemeth code to avoid switching to UEB for such punctuation.

Example 8 (punctuation/single-word switch)

EXAMPLE 3

- a. $x + 2 = 5$ and $x = 3$ are equivalent equations because both have solution set $\{3\}$.
- b. $a - 4 = 3$, $a - 2 = 5$, and $a = 7$ are equivalent equations because they all have solution set $\{7\}$.
- c. $y + 3 = 4$, $y - 8 = 7$, and $y = 1$ are equivalent equations because they all have solution set $\{1\}$.

The image shows a 10-line Braille transcription of the text in Example 3. The text is:

a. $x + 2 = 5$ and $x = 3$ are equivalent equations because both have solution set $\{3\}$.

b. $a - 4 = 3$, $a - 2 = 5$, and $a = 7$ are equivalent equations because they all have solution set $\{7\}$.

c. $y + 3 = 4$, $y - 8 = 7$, and $y = 1$ are equivalent equations because they all have solution set $\{1\}$.

Yellow highlights in the Braille indicate:

- Line 1: Opening Nemeth indicator at the end of the first line.

- Line 2: Single-word switch indicator before "and".

- Line 4: Opening Nemeth indicator for the second item.

- Line 5: Single-word switch indicator before "and".

- Line 7: Single-word switch indicator before "and".

- Line 8: Single-word switch indicator before "and".

- Line 9: Opening Nemeth indicator for the third item.

- Line 10: Closing Nemeth indicator before the final period.

Line 1: The Opening Nemeth indicator is placed at the end of text on line 1 to allow the itemized material to all start in the same cell.

Line 4-5, 7-9: Nemeth mode is not terminated. The next item begins with technical notation. The commas and periods that appear between consecutive math expressions are transcribed in Nemeth code even though they apply to the structure of the sentence.

Line 2, 5, 8: The single-word switch indicator allows the use of the contraction in "and". "And" is not being used mathematically; it is part of the sentence structure.

Line 10: Nemeth code is terminated before the period, assuming no technical notation follows this period.

Line 1: Numbers such as Figure numbers or Chapter numbers are done in UEB.

Line 3: The Opening Nemeth indicator is placed at the beginning of the displayed material because the math expression and the indicators will all fit on one line.

Line 3: Even though the commas between expressions pertain to the sentence structure, they are transcribed as Nemeth commas to avoid switching just for the punctuation.

Line 3: Material that is displayed to narrative text begins in cell 3 and runs over if needed in cell 5.

Line 3, 9: The single-word switch indicator allows us to use the contraction for "and".

Line 3, 10: The period at the end of the displayed material applies to the structure of the sentence.

Line 4-5: UEB bold word indicators are used on the bold words in the sentence.

Line 8: The Opening Nemeth indicator is placed at the end of the text preceding the displayed material because the indicators will not both fit on the same line as the expression.

Line 9-10: A mathematical expression that is displayed to narrative text begins in cell 3 and runs over in cell 5.

Line 9, 10: Commas are transcribed as Nemeth commas.

Additional Guidelines

Consistency should be maintained throughout a book. For symbols such as the percent sign, degree mark, and mentions of Greek letters, the transcriber should switch to Nemeth Code for such symbols (along with any related numbers) when the same symbol occurs in math notation elsewhere within the transcription, even though the local context does not constitute math notation. Symbols such as percent or degrees should be transcribed the same way throughout a document, even though it may mean many switch indicators.

Example 11 (percent)

In 2000, the number of drive-in movie screens in the United States was about 78% of the number in 1990. About how many drive-in screens were there in 1990?

$$\frac{78}{100} = \frac{717}{n}$$

$$78n = 100(717)$$

$$\frac{78n}{78} = \frac{100(717)}{78}$$

$$n \approx 919$$

There were about 919 drive-in screens in 1990. Check by estimating.
 78% of 919 $\approx 0.8 \times 900 = 720$, which is close to 717, the number for 2000.

1 $\frac{78}{100} = \frac{717}{n}$

2 $78n = 100(717)$

3 $\frac{78n}{78} = \frac{100(717)}{78}$

4 $n \approx 919$

5

6

7

8

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12

The Braille representation of the solution above includes several yellow boxes highlighting specific parts: the numerals 78, 100, and 717 in the first equation; the numerals 78, 100, and 717 in the second equation; the numerals 78, 78, 100, and 717 in the third equation; the numeral 919 in the fourth equation; and the numerals 78, 100, and 919 in the estimation step.

Line 1, 3, 12: The date (year) is not mathematical. The numbers are transcribed in UEB.

Line 3: The percent sign is seen again in the technical part of the paragraph; it is transcribed here in Nemeth Code. It will be transcribed the same whenever it appears in the document.

Line 4: The Opening Nemeth indicator is placed at the end of the line preceding the technical notation.

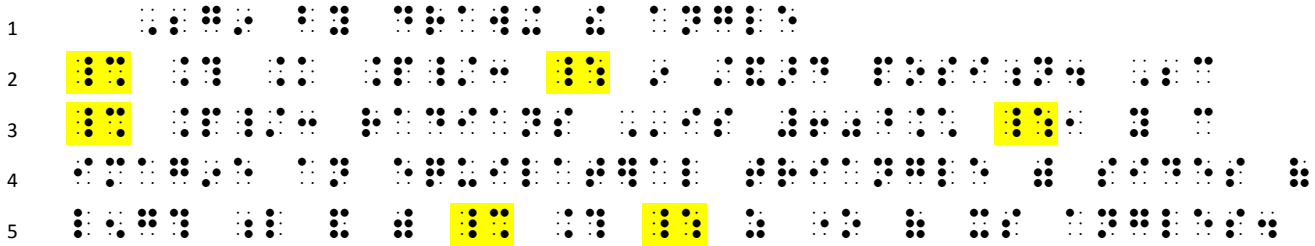
Line 5-8: A series of mathematical expressions is transcribed in Nemeth. The terminator is placed at the end of the last line of technical notation.

Line 9, 13: The numbers 717 and 919 are transcribed in UEB.

Line 11: the word "of" is part of the math expression. It is transcribed uncontracted within the Nemeth switches.

Example 12 (Greek letters, degrees)

Begin by drawing the angle $\theta = \pi/3$ in standard position. Because $\pi/3$ radians is 60° , you can imagine an equilateral triangle with sides of length l and with θ as one of its angles.



Line 2: Math notation. The entire expression including the Nemeth switch indicators will fit on one line. $\pi/3$ is a fraction. Fractions must be transcribed in Nemeth code.

Line 3: The single-word switch indicator is used for the word "is" which occurs between two technical expressions. The comma is related to the sentence structure and follows the Nemeth terminator.

Line 3: radians is the unit of measure related to $\pi/3$. Since it is part of the technical expression, no single-word switch indicator is required.

Line 3: The degree symbol is transcribed in Nemeth Code throughout the book.

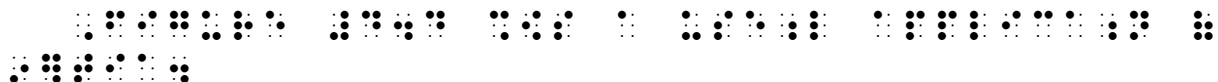
Line 5: The Greek letter θ is transcribed in Nemeth Code.

Titles for figures, tables, sections, etc. are transcribed in UEB. This applies to numbers related to the title.

Section 7-3



Figure 4.4 shows a useful application of inertia.



Measurement units (e.g., feet, ft., min) adjacent to related numbers which are transcribed in Nemeth Code are part of the technical expression and are transcribed within the Nemeth switch indicators.

Example 14 (measurement units)

94. Volume of a Cube A bottle of perfume is packaged in a box that is in the shape of a cube. Find the volume of the box if each side is 2.5 inches long.

1 **94. Volume of a Cube** A bottle of perfume is packaged in a box that is in the shape
 2 of a cube. Find the volume of the box if each side is 2.5 inches long.
 3
 4

Line 1: UEB boldface for the paragraph heading.

Line 4: Nemeth switches are used for the number containing a decimal point. Inches is a related unit enclosed within the switches.

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In a numbered or lettered series of math problems that are in Nemeth Code, leave Nemeth mode in effect for the identifiers to avoid excessive switching, even though these identifiers are not technically part of the math.

Example 15 (identifiers)

11. Physics The speed of sound through steel is 5,200 meters per second (m/s). This is 2,520 m/s faster than the speed of sound through silver. Write and solve an equation to find the speed of sound through silver.

Solve each equation.

12. $d - 4 = -7$

13. $c - 34 = 20$

14. $a - 4 = -18$

15. Astronomy Earth has an average distance of 1.496×10^8 kilometers from the Sun. If light travels 3.00×10^5 kilometers per second, how long does it take to reach Earth?

1. ⠠1⠠1⠠.⠠.⠠5⠠,⠠2⠠0⠠0⠠⠠.⠠⠠5⠠.⠠⠠⠠.⠠⠠⠠⠠.⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

2. ⠠2,⠠52⠠0⠠⠠.⠠⠠⠠.⠠⠠⠠⠠.⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

3. ⠠5,⠠20⠠0⠠⠠.⠠⠠⠠.⠠⠠⠠⠠.⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

4. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

5. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

6. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.

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8. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

9. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.

10. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.

11. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.

12.

13. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

14. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

15. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.

16. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠.

17. ⠠.⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠.⠠⠠⠠⠠⠠⠠⠠⠠⠠.

Line 1: The identifier is done in UEB. It is followed by text. UEB bold word indicator is used on **Physics**.

Line 2: The math expression extends through the "fraction" at the end of the sentence. Nemeth is terminated at the beginning of the next line. A switch indicator may stand by itself.

Line 3: Math notation is used for the expression including its related abbreviation fraction.

Line 8: The begin Nemeth indicator is placed on the line preceding the math expressions. This allows all the listed exercises to begin in the same cell.

Line 9-11: The identifiers are transcribed in Nemeth in order to save switching out of Nemeth just for those numbers. Because they are in the lower part of the cell, the punctuation indicator is required. Nemeth mode is terminated at the end of the last problem that precedes literary material.

Line 12: UEB identifier, UEB bold word indicator.

Keep the switch indicators on the same line as the mathematics to which they apply, except:

- a) At the beginning of a list of numbered or lettered identifiers, the opening indicator should be placed either by itself on the line above the first item **or** at the end of the line of preceding literary material. This ensures that all identifiers begin in the same cell. If space permits, the terminator should be placed on the same line where Nemeth mode ends. *See example 15, simbraille lines 8-11.*
- b) If space permits, an opening Nemeth Code indicator that precedes a spatial problem may be placed on the same line with the end of the text above the problem. The required blank line follows the opening indicator. If there is not room on the line with the preceding text, the indicator may be placed in cell 1 by itself and followed by the required blank line. The spatial problem is followed by a blank line (required according to Nemeth Code) and then the terminator in cell 1 on the following braille line. The blank lines are part of the spatial problem so must be inside the Nemeth switches.

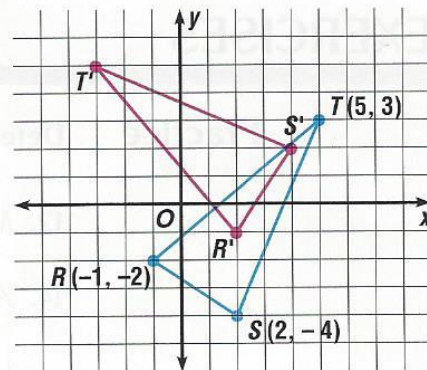
Example 16 (spatial)

Triangle RST has vertices with coordinates $R(-1, -2)$, $S(2, -4)$, and $T(5, 3)$. Find the coordinates of the vertices of this triangle after it is rotated counterclockwise 90° about the origin.

Let each column of a matrix represent an ordered pair of the triangle with the top row containing the x -values. Then multiply the coordinate matrix by the rotation matrix.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 5 \\ -2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 2 & 5 \end{bmatrix}$$

The coordinates of the vertices of the rotated triangle are $R'(2, -1)$, $S'(4, 2)$, and $T'(-3, 5)$.



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1 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

2 $\frac{1}{3}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$ $\frac{7}{14}$ $\frac{8}{16}$ $\frac{9}{18}$ $\frac{10}{20}$ $\frac{11}{22}$ $\frac{12}{24}$ $\frac{13}{26}$

3 $\frac{1}{4}$ $\frac{2}{6}$ $\frac{3}{8}$ $\frac{4}{10}$ $\frac{5}{12}$ $\frac{6}{14}$ $\frac{7}{16}$ $\frac{8}{18}$ $\frac{9}{20}$ $\frac{10}{22}$ $\frac{11}{24}$ $\frac{12}{26}$

4 $\frac{1}{6}$ $\frac{2}{9}$ $\frac{3}{12}$ $\frac{4}{15}$ $\frac{5}{18}$ $\frac{6}{21}$ $\frac{7}{24}$ $\frac{8}{27}$ $\frac{9}{30}$ $\frac{10}{33}$ $\frac{11}{36}$ $\frac{12}{39}$

5 $\frac{1}{8}$ $\frac{2}{12}$ $\frac{3}{16}$ $\frac{4}{20}$ $\frac{5}{24}$ $\frac{6}{28}$ $\frac{7}{32}$ $\frac{8}{36}$ $\frac{9}{40}$ $\frac{10}{44}$ $\frac{11}{48}$ $\frac{12}{52}$

6 $\frac{1}{10}$ $\frac{2}{15}$ $\frac{3}{20}$ $\frac{4}{25}$ $\frac{5}{30}$ $\frac{6}{35}$ $\frac{7}{40}$ $\frac{8}{45}$ $\frac{9}{50}$ $\frac{10}{55}$ $\frac{11}{60}$ $\frac{12}{65}$

7

8 $\frac{1}{12}$ $\frac{2}{18}$ $\frac{3}{24}$ $\frac{4}{30}$ $\frac{5}{36}$ $\frac{6}{42}$ $\frac{7}{48}$ $\frac{8}{54}$ $\frac{9}{60}$ $\frac{10}{66}$ $\frac{11}{72}$ $\frac{12}{78}$

9

10 $\frac{1}{15}$ $\frac{2}{22}$ $\frac{3}{30}$ $\frac{4}{38}$ $\frac{5}{46}$ $\frac{6}{54}$ $\frac{7}{62}$ $\frac{8}{70}$ $\frac{9}{78}$ $\frac{10}{86}$ $\frac{11}{94}$ $\frac{12}{102}$

11 $\frac{1}{18}$ $\frac{2}{27}$ $\frac{3}{36}$ $\frac{4}{45}$ $\frac{5}{54}$ $\frac{6}{63}$ $\frac{7}{72}$ $\frac{8}{81}$ $\frac{9}{90}$ $\frac{10}{99}$ $\frac{11}{108}$ $\frac{12}{117}$

12 $\frac{1}{20}$ $\frac{2}{30}$ $\frac{3}{40}$ $\frac{4}{50}$ $\frac{5}{60}$ $\frac{6}{70}$ $\frac{7}{80}$ $\frac{8}{90}$ $\frac{9}{100}$ $\frac{10}{110}$ $\frac{11}{120}$ $\frac{12}{130}$

13 $\frac{1}{25}$ $\frac{2}{37}$ $\frac{3}{50}$ $\frac{4}{62}$ $\frac{5}{75}$ $\frac{6}{87}$ $\frac{7}{100}$ $\frac{8}{112}$ $\frac{9}{125}$ $\frac{10}{137}$ $\frac{11}{150}$ $\frac{12}{162}$

14 $\frac{1}{30}$ $\frac{2}{45}$ $\frac{3}{60}$ $\frac{4}{75}$ $\frac{5}{90}$ $\frac{6}{105}$ $\frac{7}{120}$ $\frac{8}{135}$ $\frac{9}{150}$ $\frac{10}{165}$ $\frac{11}{180}$ $\frac{12}{195}$

15

16 $\frac{1}{36}$ $\frac{2}{54}$ $\frac{3}{72}$ $\frac{4}{90}$ $\frac{5}{108}$ $\frac{6}{126}$ $\frac{7}{144}$ $\frac{8}{162}$ $\frac{9}{180}$ $\frac{10}{198}$ $\frac{11}{216}$ $\frac{12}{234}$

17 $\frac{1}{40}$ $\frac{2}{60}$ $\frac{3}{80}$ $\frac{4}{100}$ $\frac{5}{120}$ $\frac{6}{140}$ $\frac{7}{160}$ $\frac{8}{180}$ $\frac{9}{200}$ $\frac{10}{220}$ $\frac{11}{240}$ $\frac{12}{260}$

18

19 $\frac{1}{45}$ $\frac{2}{67}$ $\frac{3}{90}$ $\frac{4}{112}$ $\frac{5}{135}$ $\frac{6}{157}$ $\frac{7}{180}$ $\frac{8}{202}$ $\frac{9}{225}$ $\frac{10}{247}$ $\frac{11}{270}$ $\frac{12}{292}$

20 $\frac{1}{50}$ $\frac{2}{75}$ $\frac{3}{100}$ $\frac{4}{125}$ $\frac{5}{150}$ $\frac{6}{175}$ $\frac{7}{200}$ $\frac{8}{225}$ $\frac{9}{250}$ $\frac{10}{275}$ $\frac{11}{300}$ $\frac{12}{325}$

21

22 $\frac{1}{60}$ $\frac{2}{90}$ $\frac{3}{120}$ $\frac{4}{150}$ $\frac{5}{180}$ $\frac{6}{210}$ $\frac{7}{240}$ $\frac{8}{270}$ $\frac{9}{300}$ $\frac{10}{330}$ $\frac{11}{360}$ $\frac{12}{390}$

23 $\frac{1}{72}$ $\frac{2}{108}$ $\frac{3}{144}$ $\frac{4}{180}$ $\frac{5}{216}$ $\frac{6}{252}$ $\frac{7}{288}$ $\frac{8}{324}$ $\frac{9}{360}$ $\frac{10}{396}$ $\frac{11}{432}$ $\frac{12}{468}$

24 $\frac{1}{84}$ $\frac{2}{126}$ $\frac{3}{168}$ $\frac{4}{210}$ $\frac{5}{252}$ $\frac{6}{294}$ $\frac{7}{336}$ $\frac{8}{378}$ $\frac{9}{420}$ $\frac{10}{462}$ $\frac{11}{504}$ $\frac{12}{546}$

25 $\frac{1}{96}$ $\frac{2}{144}$ $\frac{3}{192}$ $\frac{4}{240}$ $\frac{5}{288}$ $\frac{6}{336}$ $\frac{7}{384}$ $\frac{8}{432}$ $\frac{9}{480}$ $\frac{10}{528}$ $\frac{11}{576}$ $\frac{12}{624}$

Line 1: "Triangle RST" is the identity of the triangle. The phrase is technical material.

Line 2-3: Commas between math expressions are transcribed in Nemeth Code. A single-word switch indicator before the word "and" means the word is in UEB and is contracted.

Line 6-10: The diagram is technical and will be drawn inside the Nemeth indicators. The blank lines are part of the diagram so are expressed within the indicators. (If the diagram intervenes between two items in Nemeth mode, leave Nemeth in effect for the graphic.)

Line 14: The Opening Nemeth indicator will fit at the end of the last line of text before the spatial material.

Line 16-20: The displayed matrix begins in cell 3 with runovers in cell 5. Enlarged grouping signs enclose the matrices. The numeric indicator is required on numbers in a matrix; there is one blank cell between columns.

Line 22: The terminator is placed at the margin after the required blank line for spatial material.

If exercise directions end with an expression in Nemeth Code and the subsequent problem starts with Nemeth Code, Nemeth may be left in effect between the end of the directions and the start of the problem.

Example 17 (directions)

Find $f(x + h)$ for each function $f(x)$.

34. $f(x) = x + 1$

35. $f(x) = 2x - 3$

36. $f(x) = 4x^2$

37. $f(x) = x^2 - 2x + 5$

Line 1: The math expression is transcribed in Nemeth; Nemeth code is terminated at the end of the expression.

Line 2: Nemeth code is initiated for the technical material at the end of the instructions, but is not terminated.

Line 3: Technical material continues in the exercise. Nemeth identifiers are used so that minimal switching is required.

Line 6: Nemeth code is terminated at the end of the last problem.

When short comments in words appear alternated with math problems (such as comments on equation solutions) switch out of Nemeth code to transcribe the comments in contracted braille.

Example 18 (comments)

If y varies inversely as x , and $y = 3$ when $x = 4$, find y when $x = 18$.

$$\frac{x_1}{y_2} = \frac{x_2}{y_1}$$

$$\frac{4}{y_2} = \frac{18}{3}$$

Substitute the known values.

$$18y_2 = 12$$

Cross multiply.

$$y_2 = \frac{12}{18} \text{ or } \frac{2}{3}$$

Divide each side by 18.

The value of y when $x = 18$ is $\frac{2}{3}$.

1 ⠠⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠ ⠠⠠

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10 ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

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Comments follow on the same line as the math expression with runovers in the standard position for displayed material.

Example 20 (one-word comment)

What percent of 48 is 54?

$$n \cdot 48 = 54$$

Write an equation.

$$\frac{48n}{48} = \frac{54}{48}$$

Divide each side by 48.

$$n = 1.125$$

Simplify.

$$= 112.5\%$$

Change the decimal to a percent.

54 is 112.5% of 48.

1 $n \cdot 48 = 54$ **Write an equation.**

2 $\frac{48n}{48} = \frac{54}{48}$ **Divide each side by 48.**

3 $n = 1.125$ **Simplify.**

4 $= 112.5\%$ **Change the decimal to a percent.**

5

6 **Simplify.**

7 54 is 112.5% of 48 .

8

9 54 is 112.5% of 48 .

Line 1: The freestanding numbers are transcribed in UEB.

Line 2-4: Expressions displayed to narrative text begin in cell 3. The runover margin is cell 5. The author's comments follow the runover margin.

Line 5-8: A linked expression requiring special margins. The runover margin for the anchor and links is cell 7.

Line 6: The author's comment consists of one word. The single-word switch indicator is used on the word "Simplify" even though there are no contractions in the word.

Line 9: New paragraph of the text begins in cell 3. The words "is" and "of" are preceded by the single-word switch indicator. The Opening Nemeth indicator is placed at the beginning of the braille line since the math expression and both indicators will all fit on one line.

The bold type-form for the comments has no meaning and is not retained.

UEB type-form indicators are used in the surrounding text, and Nemeth type-form indicators are used only if it is necessary to indicate emphasis inside the code switches.

If Code switching is necessary within the text of an emphasized passage, such as in a labeled statement, the beginning type-form indicators are repeated after each switch to show that emphasis continues. The effect of a UEB type-form indicator is terminated by the switch from UEB to Nemeth. (Capitalization is not a typeform. A capitals terminator is required to end a capitals passage.) However, a switch from Nemeth to UEB does not terminate the Nemeth emphasis indicators (bold, italic, script). The appropriate type-form indicators (open and close) must be used for emphasized technical material.

To avoid excessive use of indicators, when all labeled statements in a text are printed with the same emphasis, omit the emphasis on the statement unless doing so would change the meaning. The label itself would still be fully capitalized.

The Opening Nemeth indicator and the Nemeth Terminator should be placed on the same page with part of the expression to which they apply.

Example 24 (placement of switches)

We have again used the fact that the particles are initially at rest, so that $v_{01} = v_{02} = 0$ m/s. Using this result for v_{12} , we can now obtain the final electric potential energy from the conservation-of-energy equation:

$$\begin{aligned} \text{EPE}_f &= \text{EPE}_0 - \left[\frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 \left(-\frac{m_1}{m_2} v_{f1} \right)^2 \right] \\ &= \text{EPE}_0 - \frac{1}{2} m_1 \left(1 + \frac{m_1}{m_2} \right) v_{f1}^2 \\ &= 0.150 \text{ J} - \frac{1}{2} (3.6 \times 10^{-6} \text{ kg}) \left(1 + \frac{3.6 \times 10^{-6} \text{ kg}}{6.2 \times 10^{-6} \text{ kg}} \right) (170 \text{ m/s})^2 \\ &= 0.068 \text{ J} \end{aligned}$$

20 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

21 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

22 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

23 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

24 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

25 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

----- new braille page -----

1 $\frac{1}{2}$

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6 $\frac{1}{2}$

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8 $\frac{1}{2}$

9 $\frac{1}{2}$

10 $\frac{1}{2}$

11 $\frac{1}{2}$

Line 22: The math expression will fit undivided on the line with both switches on the same line.

Line 1: The Opening Nemeth indicator should be on the same page as the expression to which it applies.

Line 2: EPE is an abbreviation for Electric Potential Energy.

Line 2-10: This is a linked expression that requires special margins: It is displayed, the comparison signs are aligned in print, and all of the links are to the right of the comparison signs. The anchor begins in cell 3, links begin in cell 5, and all runovers to anchor and links are in cell 7.

Line 3, 6: A right numeric subscript to an abbreviation requires a subscript indicator followed by a baseline indicator.

Line 7-10: A division is made between every unit in parentheses.

For a box transcribed all in Nemeth Code, the top box line may include an opening Nemeth Code indicator followed by a space at the beginning of the line. For the bottom box line, the Nemeth Terminator appears at the end of the closing box line preceded by a space. If a transcriber's note occurs inside a box that is otherwise all Nemeth Code, do not include the box lines within Nemeth Code. The UEB TN symbols must be used, and they are not part of the technical code. When possible, place the transcriber's note outside the box.

Example 26 (table)

Given the exponent, x , compute the power of 2, y .

x	$2^x = y$	y
-1	$2^{-1} = y$?
2	$2^2 = y$?
3	$2^3 = y$?
6	$2^6 = y$?

Given the power of 2, x , compute the exponent, y .

y	$2^y = x$	x
?	$2^y = \frac{1}{2}$	$\frac{1}{2}$
?	$2^y = 4$	4
?	$2^y = 8$	8
?	$2^y = 64$	64

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Line 4, 16: The entire box and its entries are in Nemeth Code. The Opening Nemeth indicator is placed at the beginning of the top box line.

Line 11, 23: The Nemeth Terminator is at the end of the bottom box line.

Example 28 (table)

[Table 1. Exposure to salinity reduces the growth of wheat plants.

Group	light	5 days	10 days
control	12	70.3±2	90±10.5
test	12	60.4±1.5	78±7.9
control	16	75.7±8	100±3
test	16	52.2±2	81±6.7

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 12

Line 3: No blank line between the title and a related box.

Line 4: Column headings are transcribed in UEB.

Line 6: The Opening Nemeth indicator is placed at the margin, indicating that what follows in the table is Nemeth code.

Line 7-8: The row headings are considered to be part of the technical material. The single-word switch indicator is not required. Contractions are not used within Nemeth indicators.

Line 11: Nemeth code terminator follows at the margin.

Line 12: The box lines are not part of the math.

Switches and Formatting

Let's take a look at applying all of these principles to a page or two from a math book, to see the guidelines in actual practice. This will involve formatting as well as enclosing the math notation in the open and terminate Nemeth switches.

When we talk about format for Nemeth, the formatting issues are those for the technical material *and* for the surrounding UEB material. The Guidance says that the entire document is arranged according to the accepted compromise between *Braille Formats: Principles of Print to Braille Transcription 2011* (for now) and the *Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision* formatting constrictions. The application of this compromise applies to the entire document, even the text parts that are not within the Nemeth switches. Dot formation is another matter. *The only time Nemeth symbols and indicators may be used is within the Nemeth switch indicators.*

Nemeth Format

Where the Nemeth Code addresses a formatting issue, this is applied first. Anything that is not addressed in the Nemeth Code will rely on Braille Formats for positioning. Some of the items governed by Nemeth Code format are:

- Division of mathematical expressions
- Narrative paragraph margins of 3-1
- Runover margins for itemized material
- Displayed math expressions
- Instructions
- Exercise with any number of subentry levels
- Table consisting only of numbers
- Blank table entries to be filled in
- Analogies
- Labeled statements
- Rules for keying

Braille Formats

- Displayed literary text
- Indented list for nontechnical text
- Headings
- Any format not covered in the Nemeth Code

There are a few things that will apply to both text and technical material. A most basic example of a Nemeth format applied to the entire document is margins for text. There are no blocked paragraphs in Nemeth Code. All paragraphs begin in cell 3 and runover in cell 1. This applies to the *Unified English Braille* (UEB) portions of text in a document in which the technical material is transcribed according to the Nemeth Code.

When simple unmodified numbers are mentioned in the text they are transcribed in UEB. However, some formatting rules of the Nemeth Code still apply. Section 195: *A hyphenated expression, of which one component is mathematical, such as "1-inch grid", may not be divided between braille lines. An abbreviation must not be placed on a different braille line from its preceding or following numeral or letter.*

Maria wants to draw a painting with an area of 40 square inches. If she drew something on 1-inch grid paper how many squares would the painting cover?

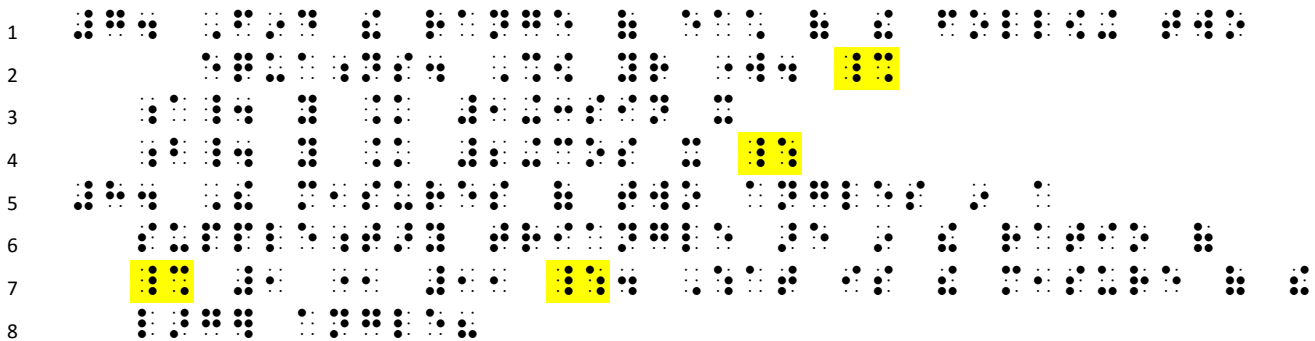
$$\frac{40 \text{ square inches}}{1 \text{ inch} \times 1 \text{ inch}} = 40 \text{ squares}$$

Martin Luther King, Jr's birthday is Jan. 15.

$$\frac{15 \text{ Jan.}}{1 \text{ inch} \times 1 \text{ inch}} = 15 \text{ squares}$$

Lists and nested lists of itemized material in exercises are formatted according to the Nemeth Code: 1-3 or 1-5, 3-5. In a list with subdivisions, the main item starts in cell 1, runovers in 5. All subdivisions, no matter their depth, begin in cell 3 and runover in cell 5. Runover location is determined individually for each item in a list. Lists of **literary** material, such as outlines, family trees, and tables of contents, are transcribed according to BF with runovers 2 cells to the right of the deepest indentation. The runover margin for an entire section of **literary** material would be the same for all items, those with subdivisions and those without.

7. Find the range of each of the following two equations. Show your work.
 - a. $y = 1 + 3 \sin x$
 - b. $y = 2 + \cos x$
8. The measures of two angles in a supplementary triangle are in the ratio of 1 : 11. What is the measure of the larger angle?



Line 1-4: Problem 7 has subdivisions. Main item is in cell 1; subdivisions begin in cell 3. The runover margin for both main item and subdivisions is cell 5.

Line 2: The Opening Nemeth indicator is placed at the end of the line of text that precedes the technical material.

Line 3, 4: The identifiers a and b are transcribed in Nemeth Code within the switches. The terminator is at the end of the last line of the math.

Line 5-8: Problem 8 has no subdivisions. The runover margin is cell 3.

Line 7: The ratio is transcribed in Nemeth Code. The open indicator, the expression, and the terminator will all fit on one line

Displayed **technical** material is transcribed according to Nemeth format.

Technical material that is displayed to *narrative text* (3-1) begins in cell 3 (two cells to the right of the runover) and runs over in cell 5. No blank lines are inserted preceding or following unless required by other rules of the Code, such as headings or spatial material.

When the material is displayed to *itemized text that contains only main divisions* (1-3), the displayed material begins in cell 5 (two cells to the right of the runover) and runs over in cell 7. When itemized material contains *both main divisions and subdivisions* (1-5, 3-5) to whatever depth, a displayed expression begins in cell 7 (two cells to the right of the runover) and runs over in cell 9.

A sentence with an equal sign is called an **equation**. Here are some examples of equations.

$$27 = 9(4 - 1) \quad 1 + 1 = 3 \quad x + 7 = 50$$

The left equation is true. The middle equation is false. The right equation may be true or false.

The image shows a Braille transcription of the text above. It consists of 9 lines. Lines 1-2 contain the introductory text. Lines 3-5 show the equations $27 = 9(4 - 1)$, $1 + 1 = 3$, and $x + 7 = 50$ displayed to narrative material. The first two equations are highlighted in yellow. Lines 6-8 show the same equations displayed to itemized text, with gray highlights indicating the main divisions and subdivisions. Line 9 contains the concluding text.

Line 3-5: The equations are displayed to narrative material. Each displayed expression begins in cell 3 (two cells to the right of the runover) with runovers, if needed, in cell 5. No blank lines are inserted.

Line 6, 7, 8: Embedded TNS explain the location of left, middle and right.

It is preferred that an exercise not be divided between braille pages whenever possible. At least one line of the instructions must be on the same braille page as the exercise. The text qualifies as an instruction if it applies to a group of numbered or lettered problems that follow. If this is not the case, the text is simple narrative material and is transcribed as a paragraph (3-1). According to Nemeth Code, margins for instructions are 5-3 with additional paragraphs beginning in cell 5 with runovers in cell 3.

Simplify.

41. $32 + 8 \div 4$

42. $24 - 2 \cdot 3 \div 6 + 1$

43. $(20 - 8) \cdot 2 + 2$

Evaluate each expression for the given value of the variable.

44. $2(4 + x) - 3$ for $x = 1$

45. $3(8 - x) - 2$ for $x = 2$

46. **Short Response** A company prints n books at a cost of \$9 per book. Write an expression to represent the total cost of printing n books. What is the total cost if 1050 books are printed?

47. **Multiple Choice** Which expression means "3 times the difference of y and 4"?

(A) $3 \cdot y - 4$


(B) $3 \cdot (y + 4)$


(C) $3 \cdot (y - 4)$


(D) $3 - (y - 4)$


Keystroke structures are technical material so must be placed within Nemeth switches.


To add whole numbers on a calculator, we use the $+$ and $=$ keys. For example, to add 57 and 34, we press 5 7 $+$ 3 4 $=$. The calculator displays 91 , so $57 + 34 = 91$. To find $314 + 259 + 478$, we press 3 1 4 $+$ 2 5 9 $+$ 4 7 8 $=$. The display reads 1051 , so $314 + 259 + 478 = 1051$.


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Line 5, 9: The window shape symbol shown is not an official part of the Nemeth keystroke rule. The symbol must be listed on the special symbols page.

Line 8-9: Keep a keystroke undivided and use as much of the braille line as possible.

Full Pages from Math Textbooks



Evaluating Algebraic Expressions

Why learn this? You can evaluate an expression to convert a temperature from degrees Celsius to degrees Fahrenheit. (See Example 3.)

An **expression** is a mathematical phrase that contains operations, numbers, and/or *variables*. A **variable** is a letter that represents a value that can change or vary. There are two types of expressions: *numerical* and *algebraic*.

A **numerical expression** does not contain variables.

An **algebraic expression** contains one or more variables.

Numerical Expressions		Algebraic Expressions	
$3 + 2$	$4(5)$	$x + 2$	$4n$
$27 - 18$	$\frac{3}{4}$	$p - r$	$\frac{x}{4}$

To **evaluate** an algebraic expression, substitute a given number for the variable. Then use the order of operations to find the value of the resulting numerical expression.

EXAMPLE 1 Evaluating Algebraic Expressions with One Variable

Evaluate each expression for the given value of the variable.

A $x + 5$ for $x = 11$

$11 + 5$ *Substitute 11 for x.*

16 *Add.*

B $2a + 3$ for $a = 4$

$2(4) + 3$ *Substitute 4 for a.*

$8 + 3$ *Multiply.*

11 *Add.*

C $4(3 + n) - 2$ for $n = 0, 1, 2$

n	Substitute	Parentheses	Multiply	Subtract
0	$4(3 + 0) - 2$	$4(3) - 2$	$12 - 2$	10
1	$4(3 + 1) - 2$	$4(4) - 2$	$16 - 2$	14
2	$4(3 + 2) - 2$	$4(5) - 2$	$20 - 2$	18

Remember!

Order of Operations

PEMDAS:

1. Parentheses
2. Exponents
3. Multiply and Divide from left to right.
4. Add and Subtract from left to right.

See Skills Bank p. SB14.

Vocabulary

expression
variable
numerical expression
algebraic expression
evaluate

1 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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3 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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6 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$ side bar

7 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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9 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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11 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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13 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

14 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

15 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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18 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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20 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

21 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

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23 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

24 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

25 $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{9}{10}$ $\frac{11}{12}$ $\frac{13}{14}$ $\frac{15}{16}$ $\frac{17}{18}$ $\frac{19}{20}$ $\frac{21}{22}$ $\frac{23}{24}$ $\frac{25}{26}$

Line 13: Underlined passage. Indented paragraph.

Line 16: Unmodified number, nonmathematical expression transcribed in UEB.

Line 18: First transcriber-defined word indicator representing highlighting. Listed on the SS page. It is not necessary to retain both the bold and the highlighting.

1		$\frac{1}{2}$	$\frac{1}{3}$	
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4		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
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6	$\frac{1}{2}$			
7	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
8	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
9	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
10	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{6}$
11	$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$			
12				
13	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{6}$
14	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{6}$
15				
16	$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$			
17		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
18				
19	$\frac{1}{2}$			
20	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
21	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
22	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	
23	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{5}{6}$
24	$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$			
25				$\frac{5}{6}$

The boxed materials follow the statements to which each applies.

Line 3-10, 16-24: The heading for each box is not mathematical. The opening Nemeth indicator is on the line preceding the technical material in the table. The terminator is placed at the end of the line which contains the final mathematical item. A blank line precedes a top box line and follows a bottom box line. Box lines are no longer (2016) required to be listed on the SS page.

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Line 7-8: Cell-5 heading

Line 9-10: Instructions, margin 5-3

Line 10: The Opening Nemeth switch is placed at the end of the line preceding the technical material so that all lettered items start in cell 1.

Line 12: Nemeth Code is terminated before the author's comment. The unmodified number and letter are transcribed in UEB.

Line 11, 14, 18: The single word switch indicator is used on the word "for" which occurs between two Nemeth expressions.

Line 19-23: A tn explains the format for a listed table.

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All of the material in the box is technical. Nemeth Code is initiated with the top box line and terminated at the end of the bottom box line.

Listed table format was selected to present the material in the print table. There are too many columns for staircase format and too many numbers for linear format.

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Line 4-9: Second transcriber-defined typeform *word* indicator is used on PEMDAS. Second transcriber-defined typeform *symbol* indicator represents the red letter at the beginning of the words in the list. Both indicators must be listed on the SS page.

Line 5-9: Sidebar material is formatted according to its text layout. This is a list with cell-5 and cell-7 headings. List items begin in cell 1.

Line 11: Cross references are 7-5 with a blank line preceding and following. Italics is not retained.

90 Chapter 1 Angles and the Trigonometric Functions

In Example 1 we found the angle when given its sine. The function whose input is the sine of an angle and whose output is the angle is the **inverse sine** function. Similarly, the **inverse cosine** and **inverse tangent** functions pair numbers with angles. Using inverse function notation from algebra, the results of Example 1 are written as

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ, \quad \cos^{-1}(1) = 0^\circ, \quad \text{and} \quad \tan^{-1}(1) = 45^\circ.$$

Note that the -1 in this notation does not indicate a reciprocal. Because there are infinitely many angles that have a given sine, cosine, or tangent, we must define these functions precisely. The precise definitions follow.

Definition: Inverse Sine, Cosine, and Tangent Functions

$\sin^{-1}(x) = \alpha$	provided	$\sin \alpha = x$	and	$-90^\circ \leq \alpha \leq 90^\circ$
$\cos^{-1}(x) = \alpha$	provided	$\cos \alpha = x$	and	$0^\circ \leq \alpha \leq 180^\circ$
$\tan^{-1}(x) = \alpha$	provided	$\tan \alpha = x$	and	$-90^\circ < \alpha < 90^\circ$

The other three trigonometric functions have inverses also, but we will not need them in studying right triangles. They are discussed in Chapter 4.

Example 2 Evaluating inverse functions

Evaluate each expression. Give the result in degrees.

a. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ b. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ c. $\tan^{-1}(\sqrt{3})$

Solution

a. Since $\cos 45^\circ = \frac{\sqrt{2}}{2}$ and $0^\circ \leq 45^\circ \leq 180^\circ$, $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$.

b. Since $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $-90^\circ \leq 60^\circ \leq 90^\circ$, $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$.

c. Since $\tan 60^\circ = \sqrt{3}$ and $-90^\circ < 60^\circ < 90^\circ$, $\tan^{-1}(\sqrt{3}) = 45^\circ$.

Now Try Exercise 5

For angles that are not multiples of 30° or 45° , we use a calculator to evaluate an inverse function. Calculators have keys labeled \sin^{-1} , \cos^{-1} , and \tan^{-1} . They are usually the second functions for the sin, cos, and tan keys. When an inverse function key is pressed, your calculator will give the angle in the range specified in the definition of the inverse functions. Of course the angle will be given in degrees or radians depending on the mode setting.

1 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

2 $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

3 $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

4 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

5 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

6 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

7 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

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9 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

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12 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

13 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

14 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

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25 $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$

Line 9, 10, 11: A list of displayed expressions.

Line 10: Single word switch indicator on the word "and".

1.3 ANGULAR AND LINEAR VELOCITY



Figure 1.22

If the speedometer in your car says 50 mph, then your velocity is 50 mph. Velocity is the rate at which the location of an object is changing with respect to time. If an object is in motion on a circle we can discuss two types of velocity, angular velocity and linear velocity.

Angular Velocity

Consider a point located on a helicopter blade that is rotating at 400 revolutions per minute as shown in Fig. 1.22. As the blade rotates, an angle is formed by the initial and terminal positions of the blade. In one revolution of the blade, the point rotates through an angle of 2π radians. The *angular velocity* of the point is the rate at which the angle is changing. Although 400 revolutions per minute could be considered an angular velocity, we will express angular velocity in radians per unit of time. To find the angular velocity we simply convert 400 rev/minute to radians/minute using $2\pi \text{ rad} = 1 \text{ rev}$ and cancellation of units:

$$\frac{400 \text{ rev}}{1 \text{ min}} = \frac{400 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{800\pi \text{ rad}}{1 \text{ min}} \approx 2513 \text{ rad/min}$$

So the angular velocity of the point is approximately 2513 rad/min. The angular velocity does not depend on the length of the blade (or the radius of the circular path), but only on the number of revolutions per unit of time.

We use the Greek letter ω (omega) to represent angular velocity, and define it as follows.

Definition: Angular Velocity

If a point is in motion on a circle through an angle of α radians in time t , then its **angular velocity** ω is given by

$$\omega = \frac{\alpha}{t}$$

Angular velocity can be expressed with many different units of time. For example, an angular velocity can be given as radians per hour, radians per week, radians per year, and so on.

Example 1 Changing the units

Convert the angular velocity 240 rad/hr to rad/min.

1
$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

2
$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

3
$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

4

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

6
$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

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$$\frac{a^2 + b^2}{c^2 + d^2} = \frac{a^2 + b^2}{c^2 + d^2}$$

Line 3-10: Linked expression not requiring special margins. Each link fills a line of braille. There are no runovers.

Line 5-7: Fractions containing cancellation must be transcribed spatially. Spatial material requires a blank line preceding and following.

Line 21-25: Labeled statement with displayed expression. Displayed to narrative margins are 3-5. No blank lines are required for displayed technical material.

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Line 1: A blank line is required following a labeled statement. If material that requires a blank line ends on line 24 or 25, the blank line should be left at the top of the next braille page.